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Allocative and Dynamic Efficiency in NBA Decision Making

Matt Goldman
UC San Diego
La Jolla, CA 92093
mrgoldman@ucsd.edu

Justin M. Rao
Yahoo! Research Labs
Santa Clara, CA 95054
jmrao@yahoo-inc.com

Abstract

This paper examines the optimality of the shooting decisions of National Basketball Association (NBA) players using a rich data set of shot outcomes. The decision to shoot is a complex problem that involves weighing the continuation value of the possession *and* the outside option of a teammate shooting. We model this as a dynamic mixed-strategy equilibrium. At each second of the shot clock, *dynamic efficiency* requires that marginal shot value exceeds the continuation value of the possession. *Allocative efficiency* is the additional requirement that at that "moment", each player in the line-up has equal marginal efficiency. To apply our abstract model to the data we make assumptions about the distribution of potential shots. We first assume nothing about the opportunity distribution and establish a strict necessary condition for optimality. Adding distributional assumptions, we establish sufficient conditions for optimality. Our results show that the "cut threshold" declines monotonically with time remaining on the shot clock and is roughly in line with dynamic efficiency. Over-shooting is found to be rare, undershooting is frequently observed by elite players. We relate our work to the usage curve literature, showing that interior players face a generally steeper efficiency trade off when creating shots.

1 Introduction

Mixed strategy Nash equilibrium (MSNE) is a workhorse in modern game theory. Decision making in a dynamic environment that involves weighing a current alternative versus the continuation value of a process of random arrivals underlies both decision theoretic problems such as search and game theoretic problems such as bilateral bargaining. In this paper we take these concepts to a difficult problem: shot choice in professional basketball. In the National Basketball Association (NBA) a 5-man line-up has 24 seconds to take the best shot possible. To do so the team must allocate the ball effectively across its factors of production (players) and over the course of the shot clock. Decisions must be made quickly and sub-optimal choice can be the difference between winning and losing. Equilibrium requires effective randomization across the 5-man line-up (*allocative efficiency*) and accurately weighing the continuation value of a possession versus the value of a current opportunity (*dynamic efficiency*). We employ rich data set comprising all shots taken in the NBA from 2006 to 2010 to fit our model of shot allocation and optimal stopping to observed shooting behavior.

Past work testing MSNE in field settings has studied 2x2 games such as soccer penalty kicks [1, 2, 3].[1] Studying a more complex game does come at a cost; it is also more challenging to fit into the confines of an abstract game-theoretic model. Our approach is to combine the stopping problem with the allocation problem. At each point on the shot clock, players must randomize across who shoots in order to maximize allocative efficiency and also

[1] The only work we are aware of that applies an optimal stopping model to professional sports is Romer (2006) [4], which examines NFL coaches' decisions to "go for it" on 4th down on the NFL. The difference is that this is a deliberative decision made perhaps 10 times per season by the coach, not the players, and the low number of occurrences means coaches might intentionally do the wrong thing in order to adhere to the "conventional wisdom".

use an appropriate threshold value for continuing the possession versus realizing a shot opportunity. The vector of shooting frequency should maximize points per possession. This vector implies a "threshold equilibrium" of play, in the spirit of a purified game [5].

The chief challenge in determining the optimality of player actions is to determine what would have happened had the player shot less or more. Our model is flexible to the fact that certain players may be able to replicate their average efficiency on marginal shots while other players may have to settle for much less efficient opportunities in order to shoot more. Just as in the theory of the firm, it is the efficiency on these marginal opportunities that should guide optimal allocation. Unfortunately, marginal opportunities are not readily observed. Understanding the relationship between the marginal and average efficiencies of individual players has long been a contentious issue in the basketball literature [6, 7, 8]. These relationships are often referred to as usage (or skill) curves and we have little hope of interpreting the optimality of observed play without addressing them. Thus a key estimation challenge we must confront, is to infer the "shot opportunity distribution" for each player. We start by assuming essentially nothing about the shot opportunity distribution in order to achieve a necessary condition of optimality. We then impose more restrictive, but still fairly flexible, assumptions in order to increase the power of our optimality tests for allocative and dynamic efficiency. The goal is to apply the strictest test of optimality without incorporating a misspecification into the abstract model.

Our key finding is that NBA players are superb optimizers. The average NBA player is shown to adopt reservation shot values *almost exactly equal to the continuation value* of his team's possession throughout the entire range of the shot clock. Very few players can be shown to individually overshoot. Undershooting is far more common and seems to occur primarily in players easily recognized as amongst the NBA elite. Such behavior is suboptimal in our simple model, but is easily rationalized if we allow for such players to conserve their energy and health for the long haul of the season at the slight expense of their team's immediate performance. Most teams core line-ups show impressive adherence to allocative efficiency, with a small spread in marginal efficiencies. We also find that NBA experience is correlated with closer adherence to optimality for both dynamic and allocative efficiency. Salary is positively correlated with departures from allocative efficiency, consistent with the idea that lower talent line-ups have less margin for error.

Additionally our model yields estimates of the slope of each player's usage curve. The estimates of any given player contain lots of noise, but collectively they conform to the conventional intuitions about which kinds of players are better at creating shots on the margin.

The paper proceeds as follows: Section 2 describes the data, Section 3 gives the model estimation and results and Section 4 concludes.

2 Data

This analysis is entirely based on play-by-play data for NBA games from 2006-2010 (four seasons). The game logs detail all the players on the court the outcome of every play. All variables discussed herein are constructed (via some fairly extensive coding) from the raw game logs.[2] Approximately 100 games (out of the 4,920 played during this time period) are missing from this data set. We have no reason to believe their omission is anything other than random. Appendix Table 1 describes the data in more detail.

3 Model and Results

In this section, we first motivate our general model of basketball and then describe the general model of optimal play. We assume risk-neutrality so that the offensive team's objective is solely to maximize points per possession. This assumption is justified due the large number of possessions per game and the central limit theorem. We also eliminate clear cases where risk-neutrality is violated, such as end of quarter and end of game situations. Additionally, this model applies only to half-court sets and not to fast breaks or actions taken immediately after

[2]Basketballgeek.com has a database of game logs, taken from NBA.com reporting.

offensive rebounds (which involve fairly trivial allocation/timing decisions). We define a fast break as any possession used within the first seven seconds of the offense and have purged all such possessions from the data. In the Appendix, we present brief justifications for both of these assumptions. "Garbage time" has been purged from our data as our model only applies to situations where the outcome of the game can reasonably be effected by chosen offensive strategy. Finally, we are unable to correctly infer the value of the shot clock for possessions occurring immediately after made field goals. The value of the shot clock is very important to our analysis, so we must throw these possessions out. After these omissions, we are left with roughly 40% of all possessions.

3.1 General model

There are 674 distinct players in our four year sample, denote each one by $i \in \{1, ..., 674\}$. $O \equiv \{i$: player i is on offense$\}$, $D \equiv \{i$: player i is on defense$\}$. Defensive strategy is summarized by the selection of five choice variables $\{d_j\}_{j \in O}$ which represent the average level of attention devoted to each offensive player across the possession. These variables are constrained by the abilities of the defenders according to:

$$g_D(d_{O_1}, d_{O_2}, d_{O_3}, d_{O_4}, d_{O_5}) \leq 0 \quad (1)$$

where g is an arbitrary function with strictly positive derivatives in all arguments.

At every one second long interval of the shot clock, each player has the opportunity to use a possession. With t seconds remaining on the shot clock ($t \in \{0, 1, ..., 17\}$), player i draws an unbiased measure of the expected number of points his team would get from his immediate use of the possession, $\eta_{i,t} \sim \Phi_{i,O/i,d_i,t}$. This value represents not just expected points on his immediate shot, but also whatever value his team is likely to get (including points scored after offensive rebounds or from foul shots) before ending their possessions. $\eta_{i,t}$ is also effected by the likelihood that player i should turn the ball over in his attempt to use the possession. $\forall i \in O$, player i has the opportunity to use the possession, getting on average a value of $\eta_{i,t}$ for his team if he does so. If no player choses to exercise the possession at interval t, than the one second long period of the shot clock is allowed to pass and each player will realize new and independent scoring opportunities in period t-1. Note that we are not assuming a viable scoring opportunity for every player at every interval of the shot clock, it is entirely possible that during many seconds of the shot clock a player will realize scoring opportunities of arbitrarily small expected value. In period 0, the shot clock is about to expire. If no player uses the possession during this period, then play is stopped and the ball is awarded to the other team with no points for the offense. In principal, the distribution of scoring opportunities could vary over player, teammates, interval of the shot clock, and the level of defensive attention. Understanding the interrelationships between all of these variables is an important direction for our future work, but for now we assume player i's opportunity distribution is invariant to teammates (O/i) and defense ($d_{i,t}$).

Definition of Variables

For some possibly singleton set $I \in O$ and a possibly singleton and convex $T \in \{0, ..., 24\}$, define $N_{I,T}$ as the number of shots used by players in I over elements of the shot clock in T and define $P_{I,T}$ as the total number of points realized from these possessions. Further, let:

$$E_{I,T} = \frac{P_{I,T}}{N_{I,T}}; T_* = \{t \in \{0, ..., 24\} : \forall t^{'} \in T, t < t^{'}\}; U_{I,T} = \frac{N_{I,T}}{N_{O,T \cup T_*}} \quad (2)$$

E is a measure of average efficiency in the periods in T and U is the hazard rate with which possessions are used over a particular interval T and by a particular subset of offensive players. Note that all of these variables describe observed data, if we wish to describe the "true" efficiency or usage rate of a particular set of players over some interval, we will use a lower case version of the same definitions (e, u).

General Solution

Each player i chooses a cut-off level, $c_{i,t}$, such that he will use the possession at interval t if and only if $\eta_{i,t} \geq c_{i,t}$. If $\forall i, \eta_{i,t} < c_{i,t}$ than no player choses to use the possession and the team proceeds to the next period of the shot clock. In order for a player's choice of cutoff to be optimal it must meet two basic criteria. *Allocative efficiency*: At any given t, the team cannot reallocate the ball to increase productivity on the margin. The frequency at which each player shoots generates equal marginal productivity. *Allocative efficiency generates the best set of shot opportunities, because randomizing effectively is a best response to selective defensive pressure*. Formally, allocative efficiency requires that $\forall i, j \in O, c_{i,t} = c_{j,t}$. *Dynamic efficiency*: Conditional on allocating efficiently in future periods, a shot is realized only if its expected value exceeds the continuation value of a possession. Formally, dynamic efficiency requires that $\forall i \in O, c_{i,t} = e_{O,t_*}$.

3.2 A Necessary Condition for Dynamic Optimality

In this section we develop a minimalistic necessary condition for dynamic optimality that makes no assumptions about our opportunity distributions. We address the question, "Do players take shots too frequently and/or too soon in the shot clock?" player i only shoots at t if $\eta_{i,t} > c_{i,t}$. Thus: $e_{i,t} = E(\eta_{i,t}|\eta_{i,t} \geq c_{i,t}) > c_{i,t}$. As such, we take as a null hypothesis of dynamic efficiency that: $e_{I,T} \geq e_{O,T_*}$. In the appendix we derive the test statistic for this hypothesis. Failing this test means a player's average output is consistently below the continuation value of the possession. We ran this test for all players in our sample with $T = \{12, ...17\}$ (the largest amount of data) and were only able to find a single player (Joel Przybilla) who violated it at $\alpha = 0.05$. These results indicate that NBA players are not taking patently wrong actions. However, we note that the power of this test is quite weak. Typically there is a divergence between a player's "worst shot" and average shot, with the average shot necessarily offering higher efficiency. Put another way, there may be much slack between their average efficiency ($e_{i,t}$) and their marginal efficiency ($c_{i,t}$). If a player's average is consistently below the continuation value, the team would be better off if he did not shoot *at all* for the interval of the shot clock (his mistakes outweigh the benefit he provides). Our results show that this optimality condition is rarely violated in NBA play.

3.3 Dynamic Efficiency Using Parametric Uniform Shot Distributions

The conditions of the previous section are quite permissive of potentially suboptimal play for two reasons: 1) they provide no tests for undershooting 2) they give no insight toward allocative efficiency. Furthermore, they offer no convenient way to characterize the overall distribution from which a player realizes scoring opportunities. In this section we address these difficulties. In order to do so we must make a somewhat restrictive assumption. Namely that $\forall t \in \{0, ..., 17\}$ player i draws his opportunities from a common distribution. This will allow us to compare player i's performance at the beginning of the shot clock (when he will rarely shoot) to the end of the shot clock (when he will have to shoot a much higher fraction of the time). Intuitively, we are using the shot clock as an instrument to identify player i's opportunity distribution and implied usage curve. Note that we have not included the final two periods of the shot clock. It is our belief that the inability to make a pass before shooting in these periods results in a fundamentally different game and different opportunity distributions. We model player i's scoring opportunities as drawn from a uniform distribution along the interval $[B_i, A_i]$. Additionally, the player selects a shot in period t if his draw lies in the interval of $[c_{i,t}, A_i]$. For the vast majority of players, B_i will take a negative value. This does not mean that players have opportunities to lose points, but merely reflects the fact that the vast majority of players do not realize a good scoring opportunity in most periods of the shot clock. As long as $c_{i,t} > 0$ the part of our theoretical uniform opportunity distribution that lies below zero is irrelevant. Conditional on the parameters $\theta_i = \{A_i, B_i, \{c_{i,t}\}_{t \in T}\}$, it is straightforward that:

$$e_{i,t} = \frac{A_i + c_{i,t}}{2}, u_{i,t} = \frac{A_i - c_{i,t}}{A_i - B_i}, \frac{de_{i,t}}{du_{i,t}} = A_i - B_i$$

$$Prob(\{E_{i,t}, U_{i,t}\}_{t \in T} | \theta_i, N_{i,t}) = \prod_{t \in T} \phi\left(\frac{U_{i,t} - \frac{A_i - c_{i,t}}{A_i - B_i}}{\sqrt{N_{i,t} \frac{A_i - c_{i,t}}{A_i - B_i}(1 - \frac{A_i - c_{i,t}}{A_i - B_i})}}\right) \phi\left(\frac{E_{i,t} - \frac{A_i + c_{i,t}}{2}}{\frac{\sigma_i}{\sqrt{N_{i,t}}}}\right)$$

The likelihood equation, is dependent upon the application of a central limit theorem to each individual period of the shot clock. Thus we proceed by performing Maximum Likelihood Estimation (MLE) for every single player with at least 15 used possessions in every relevant period of the shot clock.[3] Figure 1 provides the aggregate results of this estimation procedure. We see that indeed NBA players use a monotonically declining cut threshold consistent with the predictions of an optimal stopping problem with finite periods. More impressive is the fact that the cut-thresholds are nearly identical to the continuation values of the possession and the functions have the same shape. We present the results in two panels to enhance the contrast of the slope, while still showing it for all periods of the shot clock. Overall, Figure 1 is strong evidence in favor of near optimal play. NBA players appear to be well-tuned to the continuation value of the possession and adjust their shot choice to reflect it. This is precisely the mechanics of optimal stopping. Not only do they get the mechanics right, but the rate at which the players lower their cut threshold matches the continuation value nearly exactly!

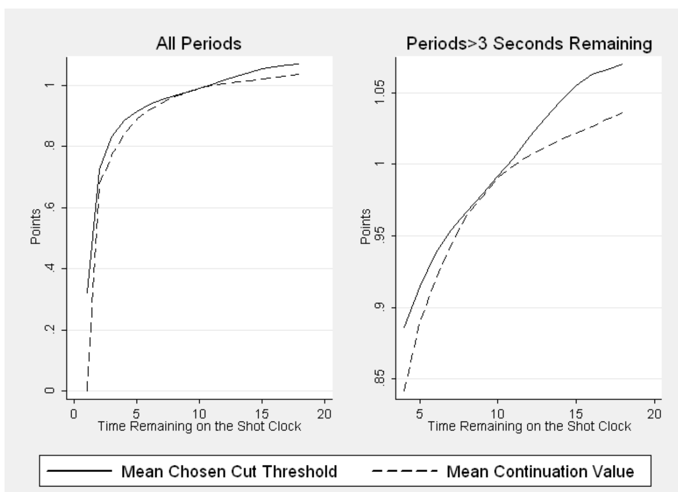

Figure 1: Estimated shooting thresholds (cut-points) and possession continuation values. Left panel: all periods of the shot clock. Right panel: restricted to periods of the shot clock with 4 or more seconds remaining to visually display the slope more clearly.

We do note, however, that in Figure 1 the cut-threshold does lie slightly above the continuation value, which is evidence that undershooting is more common that overshooting. To extend the analysis, we now examine which player's tend to overshoot or undershoot on average. We take as a null hypothesis that each player i is dynamically

[3]Despite the fact that we are fitting a uniform distribution, the support of observed efficiency and usage rates over any finite sample does not change so we do not have a regularity problem and are able to calculate standard asymptotic variances from the likelihood matrix. Additionally, we achieve parametric identification because the likelihood matrix is non-singular for all players who choose at least some variation in cutoff levels across periods of the shot clock. This should not be a surprise, if a player did the same thing in every period of the shot clock, we have no hope of learning about his usage curve. Finally, we do not observe the true model lies within our specification and apply consistently estimated covariance matrices to our results from White (1982).

efficient on average. Namely that for each player i, $\sum_{t\in\{0,..,17\}} \omega_t c_{i,t} = \sum_{t\in\{0,..,17\}} \omega_t e_{O,t_*}$. To maximize power to detect deviation, we test this hypothesis with a weights inverse to the variance of our estimated cutoffs. Figure 2 provides a histogram of the resulting t-statistics. A negative t-statistic indicates overshooting, a positive t-statistic indicates undershooting.

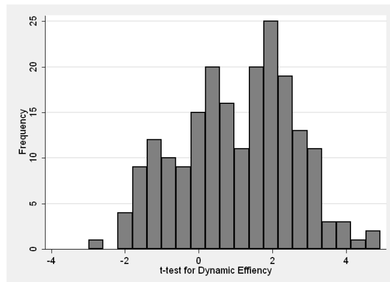

Figure 2: Player-by-player t-statistic for deviations from dynamic optimality. Positive values indicate "under-shooting."

The results again indicate that undershooting is much more common than overshooting. In fact, only 5 players are found to be significant over-shooters — less than we would expect to find by chance alone (although if we take the mean to be 1, not 0, the evidence these players overshoot strengthens considerably.[4] In line with this reasoning, the distribution appears standard normal but shifted over about 1 (the mean 0.98) Most players appear to be optimizing and mistakes tend to come in the form of undershooting early in the shot clock. In this sense, some players wait too long to shoot or do not expend maximum effort on each possession. In contrast, lab subjects tend to pull the trigger too early, typically through the use of a fixed threshold [9]. We also compute the loss in surplus due to sub-optimal shooting decisions (intuitively integrating between the two lines in Figure 1). The median value of DWL across players is 4%, consistent with nearly optimal play.

To investigate which factors lead to under/over-shooting we regress the t-stat from adherence to dynamic optimality on individual player characteristics and find that salary is positively related to the t-stat (coef 0.08 per M, $t = 2.75, p < .007$). Indeed the league's star players such as Chris Paul, Lebron James and Kobe Bryant have high t's. The fact that higher paid players are more likely to $under$ shoot is perhaps surprising at first. For instance, some readers might have the intuition that NBA players interests diverge from team interests in that they have the incentive to raise their point average through suboptimal play. Under this view, the labor market rewards the wrong attributes (points per game as opposed to efficiency, for example). Our results are inconsistent with the view of labor market. Boosting individual production at a cost to the team does not seem to be a strategy frequently employed by NBA players. We think this is interesting in its own right. Teams still have a principle-agent problem in that long-term contracts create a moral hazard for effort, but it is interesting that very few players exhibit "selfish play." Our belief is that the prevalence of under-shooting among the higher paid players is evidence that the better players conserve energy at times due to their high playing time and long season (over-shooting would be impossible to rationalize this way and we observe *far* less over-shooting).

3.4 Allocative Efficiency

Allocative Efficiency is the hypothesis that the players on the court, (I) use the same cut-point in all periods of the shot clock. This implies they have equal marginal efficiencies; put another way, it means the team cannot reallocate the ball to relatively more productive players on the margin and boost output per possession. In our estimation we allow the the set I to be an entire five man lineup or a three or four man "core" that more frequently shares the

[4] Lamar Odom, Monta Ellis, Rafer Alston, Russell Westbrook and Tyrus Thomas are the guilty parties.

court. The concept of cores is convenient to improve power in estimation Because our estimation is more precise for these periods, we will focus on cut-points in the first thirteen seconds of the shot clock ($T = \{5...18\}$). Let c be the $\|T\|\|I\| \times 1$ vector of relevant cut points, sorted first by period of the shot clock. Based on our parametric procedure we have: $\hat{c} \sim N(c, \frac{V_c}{N})$. We define the "true" deviation from Allocative Efficiency as $spread$.

$$S_{I,T} = \sum_{t \in T} \sum_{i \in I} (c_{i,t} - \bar{c}_t)^2 = (c - \bar{c})' I (c - \bar{c}) = z' I z$$

It turns out that the natural empirical analog to this measure is biased upwards in small samples, we show in the Appendix that we can derive an unbiased estimator by making a relatively straightforward bias correction. We use the de-biased measure of spread through the paper. In this section our analysis focuses on 3-man cores. Using complete 5 man line-ups is untenable from a sample size perspective, the results from 3-man and 4-man cores do not meaningfully differ, however the data is about 5 times larger for 3-man cores. We include in our analysis any core of 3 players for which each player $i$in the line-up takes at least 15 shots for every interval of the shot clock.

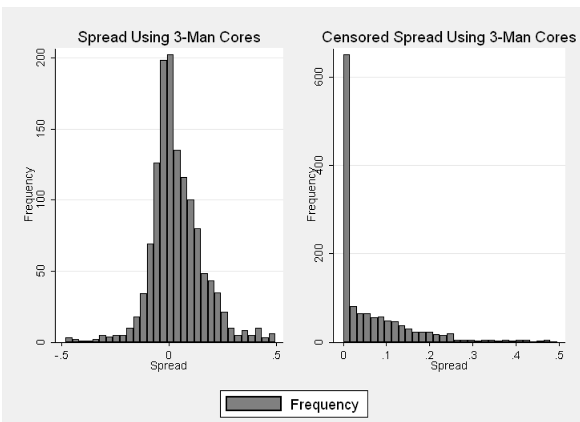

Figure 3: Distribution of $spread$ for 4-man cores. Higher values indicate larger deviations from optimality.

Table 1: Line-up characteristics and $spread$

Offensive Efficiency	-0.0204
	(0.0541)
Possessions	1.08e-05***
	(2.57e-06)
Mean Salary	0.00559***
	(0.00207)
S.D. Salary	1.38e-09
	(2.22e-09)
Mean NBA Experience	-0.00840***
	(0.00213)
S.D. NBA Experience	0.000130
	(0.00323)
Observations	1,307

*** p<0.01

Figure 3 presents the distribution of $spread$ for the 1338 qualifying 3-man cores in our data. The left panel presents the raw measure; the right panel censors negative values to 0, this is done to show massive spike in line-ups that we estimate to be achieving allocative efficiency. The median is exceedingly small; it comes in at 0.017 (the mean is slightly larger at 0.037). Given 100% adherence to allocative efficiency, we would expect these values both to be 0. Even though the deviations we see are small in absolute magnitude, we can strongly reject that the median (sign rank $z = 6.61$) and mean (t-test $t = 8.05$) are 0. This evidence the power of our test and indicates that the large spike at zero is a true finding of allocative efficiency — in most line-ups, the players show near equal marginal efficiencies at each period of the shot clock. Again, this is strong evidence of near optimal play, but again play is not perfect. Players seems to be sharing the ball incredibly well, but still about 10% of line-ups are estimated to have spread in excess of 0.19. These line-ups show clear room for improvement in terms of ball allocation. To investigate further, we regress $spread$ on a few line-up factors in table 1. We note that there are serious endogeneity issues in studying the relationship with offensive efficiency because a coach will likely let a productive group of bad-sharers continue to play together, even though allocative efficiency would improve their performance. Experience has a strong, significant impact in reducing spread, indicating either that player's improve shot selection with age or the existence of a selection effect where better optimizers last longer in the league. Finally, higher paid line-ups spread the ball less effectively. This is consistent with the idea that line-ups with lower overall ability have to allocate the ball well to receive significant playing time.

3.5 Variation in the Usage Curve by Position

As noted above our parametric model also predicted the slope of the usage curve for each player as the length of their uniform opportunity distribution ($\frac{A_i - B_i}{2}$). These predictions were noisy for individual players, but via a secondary regression we can observe patterns in how the usage curve's of individual players are related to their primary position and their role within their offense (usage% is defined as the fraction of a team's possessions a given player uses). We use the standard convention for position numbering (1: point guard, 2: shooting guard, 3: small forward, 4: power forward, 5: center.

Table 2: Robust OLS Regression Explaining the Usage Curve

LHS/RHS	Point Guard	Shooting Guard	Small Forward	Power Forward	Center	Usage %
$\hat{A_i - B_i}$	5.02***	5.73***	5.61 ***	6.32***	6.52 ***	-8.5***
s.e.	(0.52)	(0.59)	(0.58)	(0.62)	(0.65)	(1.94)

$R^2 = 0.8465$, *** p<0.01

All t-statistics are very significant and coefficients on the position variables increase in a statistically significant way (p=0.0003). The results indicate that larger players (moving towards position 5) have steeper usage curves. We argue that this result should be intuitive. Especially skilled ball handlers and shooters (primarily found at the 1 and the 2) should have little difficulty creating an additional shot under the pressure of the shot clock. However, larger players will receive many easy inframarginal opportunities by virtue of catching the ball close to the basket, but might have more difficulty creating additional shots in less advantageous positions. Additionally there is a very significant effect whereby players who use a higher percentage of their team's shots tend to have a flatter usage curve. This is strong evidence that NBA teams are aware of heterogeneity in their player's abilities s to create shots and tend to allocate more shots to player's who are better at it. We note that the fitted values for the slope of our usage curve vary from roughly 2.5 (for a very high usage point guard) to almost 6 (for a low usage center). A typical player who wants to use an additional one percentage point of his teams shots, must raise his hazard rates by roughly .002 in all periods and will then experience a drop in efficiency between .0025 and .005 depending on his characteristics.[5] It is numbers resulting from this calculation that should be compared to other estimates of the slope of the usage curve.

4 Discussion and Conclusion

This study uses a huge volume of quick decisions made by players observing a random arrival of shooting opportunities. We model shot selection and allocation in the game of basketball and examine the behavior of professionals relative to theoretical standards of optimality we derive. The decision to shoot is modeled as a dynamic allocative stopping problem. By using a realistic modeling approach we are able to derive strict tests of optimality. We find that players overall adhere quite closely to the theoretical predictions; overall they are suburb optimizers, although mistakes are made. In the context of dynamic efficiency, the shot threshold has precisely the correct slope and nearly overlaps the continuation value of the possession. The mistakes that are made tend to be undershooting, in that the continuation value is lower than the marginal shot; these "mistakes" could be rationalized by the conservation of energy across possessions. In allocative decision making, most teams show a very low variance of marginal efficiencies across players on the court. For both measures, NBA experience is correlated with closer adherence to optimality. Finally, we presented new and powerful estimates of player specific usage curves that conformed nicely to conventional intuitions about shot creation.

[5] We only estimate the usage curve for non-rotation players. Rotation players presumably have steeper usage curves (given their role on the team), this probably explains why our estimates are a touch lower than "conventional wisdom."

5 Acknowledgments

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6 Appendix

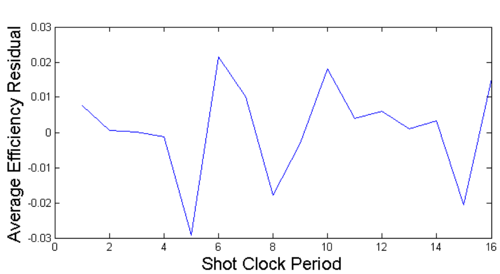
6.1 Tables

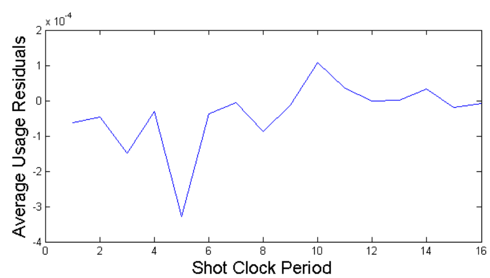
Appendix Table 1: Data Overview and Description

Event/Action	Description
Offensive/defensive line-up	Players on court at given time
Game-time	Minutes and second of each event
Game day	Date of game
Shooter	Player/time of the action
Rebound/assist	Player/time of the action
Foul	Shooting, non-shooting, flagrant, illegal defense
x,y coordinate of shot	Physical location of shot
Turnover	Broken down by bad pass, dribbling error, charge, lost ball

6.2 Parametric Model Specification Tests

Our model is identified by assuming invariance of a player's ability to realize scoring opportunities across different values of the shot clock. However, because we make this assumption for 16 different periods of the shot clock, our model is overidentified and our assumptions can be tested. Suppose for example that defenses became progressively more tenacious over the course of the shot clock and that player's opportunity distributions generally declined across the shot clock. Then, we would find that a single distribution could not accurately reflect a players ability to score in both the beginning and end of the shot clock. Players would end up shooting less and less efficiently towards the end of the shot clock than our model would predict. For now, we present plots showing how our model preforms across the shot clock as an average across all players in our sample. We take solace in the lack of any obvious trends in our residuals.





6.3 Necessary condition derivation

A convenient test is implied by application of a central limit theorem, to give:

$$E_{I,T} \sim N(e_{I,T}, \frac{\sigma_I}{N_{I,T}})$$

$$E_{O,T_*} \sim N(e_{O,T_*}, \frac{\sigma_O}{N_{O,T_*}})$$

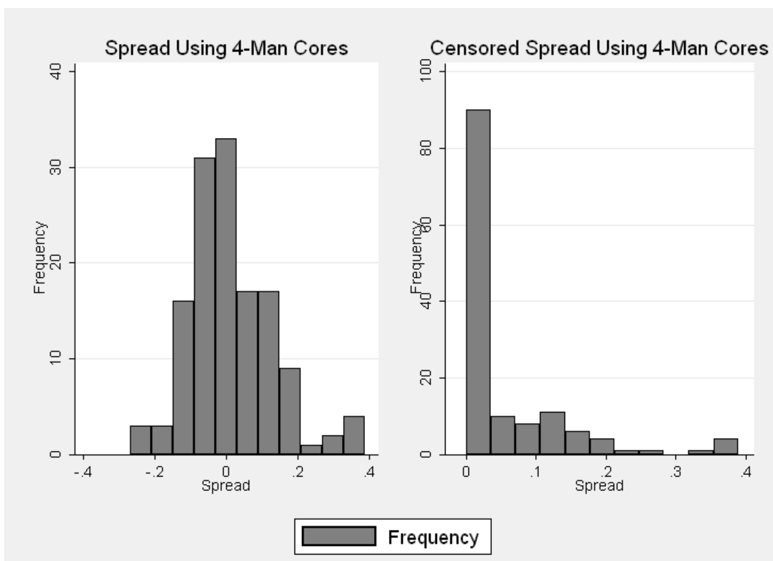
$$T \cap T_* = \emptyset \to E_{O,T_*} \perp E_{I,T}$$

$$E_{I,T} - E_{O,T_*} \sim N(e_{I,T} - e_{O,T_*}, \sqrt{(\frac{\sigma_O}{N_{O,T_*}})^2 + (\frac{\sigma_I}{N_{I,T}})^2}$$

6.4 Unbiased measure of spread

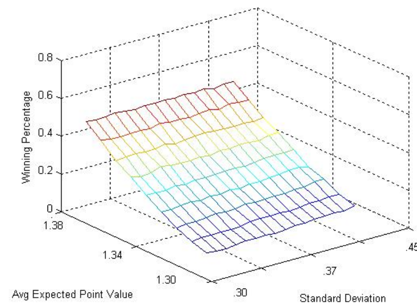
We define the "true" deviation from Allocative Efficiency as $spread$.
$S_{I,T} = \sum_{t \in T} \sum_{i \in I} (c_{i,t} - \bar{c}_t)^2 = (c - \bar{c})' I (c - \bar{c}) = z' I z$
Where $c_{i,t}$ is the cutoff chosen by player i in period t and $\bar{c}_t$ is the average over the five teammates of the cutoff chosen in period t and $\bar{c}$ is the appropriate corresponding $\|T\|\|I\| \times 1$ vector of averages.
Note that, if we define $M = I_T \otimes (I_5 - \frac{J_5}{5})$, we may also simply write $z = Mc$ and $S = c'M'Mc = c'Mc$.
The "seemingly" natural empirical analog to S is:
$\hat{S} = \sum_{t \in T} \sum_{i=1}^{5} (\hat{c_{i,t}} - \hat{\bar{c}_t})^2 = \hat{z}' I \hat{z}$
However, a little bit of calculation reveals that this measure is biased upwards, especially for small samples.
$E(\hat{z}) = c - \bar{c}$
$V(\hat{z}) \equiv V_z = \frac{MV_cM}{N}$
Thus: $E(\hat{S}) = \sum E(z_{i,t}^2) = \sum E(\hat{z_{i,t}})^2 + V(\hat{z_{i,t}}) = S + \sum diagV_z = S + \frac{\sum diag(MV_cM)}{N}$
turns out to be upward biased and especially so for small n. In order to correct for this, we essentially subtract off the bias and define:
$\hat{S^*} = \hat{S} - \frac{\sum diag(M\hat{V_c}M)}{N}$

6.5 4-man Cores



6.6 Additional support for Risk Neutrality

The following plot shows the simulated winning percentage for an underdog with baseline mean expected point value of 1.38 playing a team who averages 1.4 points per possession with standard deviation 0.45. Each "game" was simulated 10,000 times. As evidenced by the figure, although the underdog wants to increase the standard deviation of the expected value of shot attempts it does not want to trade off any meaningful amount of mean to do so.



Appendix Figure 1: Underdog winning percentage as a function of standard deviation and mean

6.7 When Does Half-Court Offense Begin?

We decided that half-court offense began with 17 seconds on the shot clock. Our reason for doing so, is that prior to 17 seconds the average value of exercising a possession is found to be strongly correlated with the mechanism by which the possession originated (steal, dead ball, or defensive rebound). However, from 17 seconds on team's are in a half court set and the average value of possession use is now independent of how the possession originated.